1 The mathematics

1.1 B-spline definition

$$n = \text{degree of spline} \tag{1}$$

$$\tau = \text{number of knots} \tag{2}$$

$$\pi = \text{number of control points} \tag{3}$$

$$t_0 \dots t_{\tau-1} = \text{knot values} \tag{4}$$

$$\mathbf{P}_0 \dots \mathbf{P}_{\pi-1} = \text{control points} \tag{5}$$

1.2 Algorithm

$$t_b \le t < t_{b+1} \Rightarrow \mathbf{S}(t) = \mathbf{Q}_b^n \tag{6}$$

$$\mathbf{Q}_k^0 = \mathbf{P}_k \tag{7}$$

$$\mathbf{Q}_k^d = (1 - \alpha_k^d) \cdot \mathbf{Q}_{k-1}^{d-1} + \alpha_k^d \cdot \mathbf{Q}_k^{d-1} \tag{8}$$

$$\alpha_k^d = \frac{t - t_k}{t_{(k+1)+(n-d)} - t_k} \tag{9}$$

2 Procedure

1. Let $T = [t_0 \dots t_{\tau-1}]$.
2. $(T_L, T_H) = \text{span } (t \ge)\ T = ([t_0 \dots t_b], [t_{b+1} \dots t_{\tau-1}])$.
3. $T_l = $ last n values from $T_L = [t_{b-n+1} \dots t_b]$.
4. $T_h = $ first n values from $T_H = [t_{b+1} \dots t_{t+n}]$.
5. $b + 1 = $ length of T_L.
6. Let $P = [\mathbf{P}_{b-n} \dots \mathbf{P}_b]$.
7. $Q = \text{zip } (\mathbf{P}_l \leftarrow P, \mathbf{P}_h \leftarrow \text{tail } P, t_l \leftarrow T_l, t_h \leftarrow T_h)$:
   (a) $\alpha = \frac{t - t_l}{t_h - t_l}$
   (b) $\mathbf{Q} = (1 - \alpha)\mathbf{P}_l + \alpha\mathbf{P}_h$
8. If Q contains only 1 item, return it. Otherwise...
9. Let $P = Q$.
10. Let $T_l = T_l$ with first value removed.
11. Let $T_h = T_h$ with last value removed.
12. Go to step 7.